8. Hubble constant and time change

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If we start from the approach of a linearly growing universe, then we have two possibilities, to solve the Einstein field equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = -\frac{8\pi G}{c^2} T^{\mu\nu} \qquad (1) \ .$$

On the one hand, we can use the approach of a static, spherically symmetrical body, in which the mass of the matter increases constantly with the radius and we take the interior solution. On the other hand, we can continue to use a dynamic approach and assume the universe to be isotropic. Then our position is approximately in the middle range and we could consider the space as homogeneous, as the energy density is no longer evenly distributed, but the space between the bodies can move homogeneously over the whole universe. We can then take a closer look at this movement with this special mass distribution. This means that the space can move, but it should not move anymore and therefore it does not matter whether the movement looks the same everywhere. Interesting with this approach is, that we can understand the scale factor R(t), which is introduced thereby, also as a time quantity. If not the space, but the time changes with the radius, then the objects also distance themselves from each other, however now no longer spatially, but temporally. If, for example, time passes more slowly at a greater distance than it does for us, a human being would age more slowly there and this difference would be all, the more noticeable, the longer time passes. We move away from each other in time. Time is no longer constant but position-dependent. Further inside it passes faster, further outside slower. So the time velocity \dot{T} decreases towards the outside. This decrease should be proportional to the distance and thus proportional to the scale factor R(t). Since it is used here as time factor, the time velocity would go slower with increasing time duration, which the signal needs to come to us, and that proportionally T/T = const (2). If we are approximately at the origin, then any signal that geodetically comes to us would have a slower time velocity. This proportionality would lead to a redshift that cannot be distinguished from an apparent escape velocity of the

galaxies. A movement with a constant velocity means a time expansion and a change in three-dimensional space in one of the three spatial directions. Movement equals time stretching, but also accelerated systems stretch time. There are accelerated systems like those in the gravitational field of masses, which rest on the surface of the bodies by electrical counterforces. They are in the same system and do not move. Nevertheless, time is slower there. So systems that are balanced by counterforces can stretch time without moving in space.

On earth, this is possible on the solid ground. The gravitational forces are very strong with such large masses and the electrical forces are the only ones that can build up a counterforce. When looking at large dimensions in the universe, only attracting forces act on all masses, since the bodies as a whole are electrically neutral. Consequently, all masses must attract each other over short or long distances.

In our structure this is different. Here the two planes, from which the elementary particles are composed, can shift to each other. If the two inner planes come closer to each other, then the mass rises, they absorb energy. Conversely, if they move away from each other they release energy. From the point of view of the atoms, the distance to other particles in this connecting dimension becomes somewhat smaller or larger. Time and space are stretched a little. For this the particle does not necessarily have to move towards the second particle, it can also only move more strongly with a superimposed counterforce, for example in disordered trembling motions. However, this is not the time-determining quantity that is meant here. Beside the spatial distances, which can be measured or determined in coordinate systems, there is a second process, which gives life to the whole process. A time process, that is a time quantity that determines how often two spatially distant particles exchanges per time unit. This exchange process then also depends on the spin. The planes change in three cycles, whereby each cycle section always has the same time measure. This rotation speed determines the time course, because it is the time quantity. It can go slower and faster. The time change \dot{T} can be slowed down by a movement in space, but also by other masses, thus walking slower. For all particles, than this time cycle is slower, so that a human in a fast flying rocket, does not notice it at all, because this time expansion affects all time processes of all particles moving with it.

Transferred to the curved universe, this means that particles can release energy in order to rise in the radius direction of the universe or one would have to add energy in the form of a change in time if one wanted to get further inwards. Both directions are exactly in equilibrium with the force of attraction. Their average speed is zero, but time passes outside more slowly than further inward and that roughly proportionally to the distance. The redshift, therefore, could mean that the galaxies move away from us, but then an abstract space would have to expand. It could also mean that the further we are from the center, the slower time will pass. Since we are dealing with new particles at the edge, which only slowly establish contact with each other, the particles feel the gravitational forces only belatedly.

If we first take the first case of a static spherically symmetrical mass distribution and consider the timeindependent solutions, then the following applies to the general approach of the line element

$$ds^{2} = e^{v(r,t)}c^{2}dt^{2} - e^{\lambda(r,t)}dr^{2} - r^{2}(d\mathcal{P}^{2} + \sin^{2}\mathcal{P}d\varphi) \quad (3)$$

v(r,t)=v(r) and $\lambda(r,t)=\lambda(r)$ should apply because it is spherically symmetrical on the one hand and static on the other. This reduces the line element to

$$ds^{2} = e^{v(r)}c^{2}dt^{2} - e^{\lambda(r)}dr^{2} - r^{2}(d\mathcal{P}^{2} + \sin^{2}\mathcal{P}d\varphi) \quad (4) .$$

For the energy pulse tensor we also take the approach for gases and liquids with isotropic pressure. The pressure p and the density ρ should be time-independent and spherically

symmetrical:
$$T^{\mu\nu} = (\rho_0 + \frac{p}{c^2})U^{\mu}U^{\nu} - pg^{\mu\nu}$$
 (5)

We assume that matter does not move, then is $\frac{dx'}{d\tau} = 0$, so are all U' = 0 and $U^0 = dx^0/d\tau$.

It applies $U^0 = g_{00}^{-1/2}$ and $U^0 = g_{00}^{1/2}$ from this follows $U^0 = g_{00}^{-1/2}$ and so $g_{00} = e^{v(r)}$ and we get $U_\lambda = (e^v c, 0, 0, 0)$.

With (5) follows
$$T_{00} = g_{00}(\rho c^2 + p) - g_{00}p = e^{\nu}\rho c^2$$
 (6).

Further we get from zero different terms for

$$(T_{11}, T_{22}, T_{33}) = p(e^{\lambda}, r^2, r^2 \sin^2 \theta) \quad (7) .$$

With $T = g^{\lambda\mu}T_{\lambda\mu} = \rho c^2 - 3p$ (8), for the tensor $S_{\mu\nu} = T_{\mu\nu} - (g_{\mu\nu}/2)T$ (9) with the only components other than zero, the following results are obtained

$$(S_{00}, S_{11}, S_{22}, S_{33}) = \frac{1}{2} \Big[(\rho c^2 + 3p) e^{\nu}, (\rho c^2 + p) r^2, (\rho c^2 + p) r^2 \sin^2 \vartheta \Big] \quad (10) .$$

Inserted into the field equation follows with the condition that all partial derivatives after time are zero

$$v'' + \frac{1}{2}v^2 - \frac{1}{2}\lambda'v' + \frac{2}{r}v' = \frac{8\pi G}{c^4}(\rho c^2 + 3p)e^{\lambda} \quad (11)$$
$$v'' + \frac{1}{2}v'^2 - \frac{1}{2}\lambda'v' + \frac{2}{r}\lambda' = \frac{8\pi G}{c^4}(\rho c^2 - p)e^{\lambda} \quad (12)$$
$$v' - \lambda' = \frac{2(e^{\lambda} - 1)}{r} - \frac{8\pi G}{c^2}(\rho c^2 - p)re^{\lambda} \quad (13) .$$

These are the three differential equations that are used in a static, spherically symmetrical structure with a density that does not disappear and an isotropic pressure. From this, the still unknown ν and λ proudly presents. If you subtract equation (12) from equation (11) and solve $(re^{-\lambda})'$, then after the integration we get

$$e^{\lambda} = \left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1} \quad (14)$$

and thus for $v' = \frac{2G[M(r) + 5\pi r^3 p/c^2]}{c^2 r^2 [1 - 2GM/(c^2 r)]}$ (15). The integrated from r > R to $r = \infty$ and the condition that the metric should become pseudo-Euclidean in the infinite, leads us to $v(r) = \ln\left(1 - \frac{2GM}{c^2 r}\right)$ (16). From this follows the equation known as the Oppenheimer-Tolman-Volkoff equation

$$p'(r) = -\frac{GM\rho[1+p/\rho c^2][1+4\pi r^3\rho/(c^2M)]}{r^2[1-2GM/(c^2r)]} \quad (17) ,$$

which we here transfer to the entire universe.

So far it has been shown, that with a spherically symmetric approach, where we are close to the center, there are no changes in the approach. Since, however, it is not to be expected that at the typical density in a universe evenly filled with matter a considerable pressure would build up, we would not have to expect a state of equilibrium, except in stars.

The original approach was to apply the field equations to suns up to neutron stars. The inner pressure by the atoms and molecules opposes the gravitational attraction and leads to an equilibrium condition, which results in a static construction. If we want to transfer this approach to the extremely thinned out whole space of the universe, then the hydrostatic pressure can be completely neglected here. The particles experience only a weak attraction to the center, but this is not stopped in any way by a particle motion pressure.

However it is shown, under our special conditions something like counter-pressure is build up, because mass belongs to certain universe sphere and we can move particle only in Rdirection towards center, if we supply energy. This energy is of the same size as the gravitational attraction, but with opposite signs. Related to a single particle is the energy change in R-direction, which is needed to raise the particle

from R₁ to one R_e-step further,
$$\Delta E = (m_t - m_t)c^2 = c^2 M_{U_0} \left(\frac{R_e}{R_1} - \frac{R_e}{R_1 + R_e}\right)$$

(18). The potential, in turn, that increases when you move one $R_{\rm e}\mbox{-step}$ further away from the center is

$$\Delta E_{Pot} = \frac{GM(R_1)m_t}{R_1 + R_e} - \frac{GM(R_1)m_t}{R_1} = \frac{GM_{U_0}R_1m_t}{R_e} \left(\frac{1}{R_1 + R_e} - \frac{1}{R_1}\right) \quad (19) \ .$$

With the basic condition that applies to R₁ itself $\frac{GM(R_{\rm l})}{R_{\rm l}c^2}=1$

and the condition $m_t = M_{U_0} \frac{R_e}{R_1}$ (20) both energies are equal but of opposite signs.

A further point concerns the structure of the elementary particles themselves, which we here assume with two planes of the size $A = R_e^2$ and whose distance, with increasing universe radius, more and more moves away from each other. Here one could go one step further and understand the electric field of the two planes and their distance to each other, as a kind of pressure, which is in equilibrium to the potential energy on the respective universe shell. Starting from the distance R_e , an increasing approach of the planes means that energy has to be applied, against the electric field. We had already found out in other context that the ratio with which an electron and a proton attract each other is equal to the ratio of the basic mass M_{U0} to the respective particle mass. For us this would be the mass of the proton

 $\frac{F_{el}}{F_G} = \frac{e^2}{4\pi\varepsilon_0 \cdot G \cdot M_e M_t} = \frac{M_{U_0}}{M_t} \quad (21). \text{ In addition, the ratio should again}$

apply to the plane spacing and its masses $\frac{d_t}{R_e} = \sqrt{\frac{M_e}{M_t}}$ (22). If we

use this for the electrical force and transform it, we obtain the relationship

$$F_{el} = \frac{e^2}{4\pi\varepsilon_0 d_t^2} = \frac{GM_{U_0}M_e}{d_t^2} = \frac{GM_U(R_1)M_t}{R_1R_e}$$
(23)

Since we have used the force field for spherical symmetrical charges above, we must also use 4π for the plane distance accordingly. Thus the size with which the planes in the proton are pushed apart is the same size, as the force of the entire mass up to the location R_1 with which a single particle is attracted.

If we now come back to the TOV equation and consider the pressure change for a universe where the mass at the edge increases linearly, then we receive

$$P'(r) = -\frac{\frac{4\pi}{3}G\rho_0 r^3 \left(1 + \frac{P}{\rho c^2}\right) \left(1 + \frac{3P}{\rho c^2}\right)}{r^2 \left(1 + \frac{2GM}{c^2 r}\right)} \quad (24) \ .$$

The denominator disappears for $r = R_1$, so the pressure change would be infinite unless one of the counter terms also disappears. Since we assume a static universe it should be $\left(1+\frac{P}{\rho c^2}\right)=0$ or $\left(1+\frac{3P}{\rho c^2}\right)=0$ and it should apply $P(r)=-\rho(r)c^2=-\frac{M_U(r)c^2}{V(r)}$ (25) for the pressure.

Let us reshape the energy that is in each elementary particle in the plate spacing and calculate the total energy that results from it

$$P(R_1) = \frac{E(R_1)}{V(R_1)} = -\frac{e^2 R_e}{4\pi\varepsilon_0 d_t^2} \frac{R_1}{R_e} \frac{M_{U_0}}{M_t} \frac{1}{V(R_1)} \quad (26) .$$

In it $\frac{R_1}{R_e} = \frac{M_{U_0}}{M_t}$ stands for the total number of particles up to

R₁. If we use equation (21) for $\frac{e^2}{4\pi\varepsilon_0}$, then together with (22)

we obtain
$$P(R_1) = -\frac{GM_{U_0}M_eR_e}{d_t^2}\frac{M_U(R_1)}{M_t}\frac{1}{V} = -\frac{GM_{U0}M_eR_eM_tM_U(R_1)}{R_e^2M_eM_t} = -c^2\frac{M_U(R_1)}{V(R_1)}$$

This means that the force, with which the two charged planes are compressed, represents a pressure or an energy density. It is on the one hand in equilibrium to the gravitational potential of the universe-shell as a whole. On the other hand, the pressure of all these particles is always equal to the gravitational energy density. And thus the electrical energy density of the elementary particles compensates the gravitational attraction. The basic structure would therefore be static.

Since in (24) the density can no longer be assumed to be spatially constant over the whole universe, we write it down as a function of the position to $\rho(r) = \frac{M_U(r)}{V(r)}$. There also the total mass changes with the radius.

We got a pressure change at position r, which is influenced by the whole universe, the total mass within the r-shell and which depends on the total density.

$$P'(r) = -\frac{\frac{GM_{U_0}}{R_e}r\left(1 + \frac{P_{el}}{c^2M_U/V}\right)\left(1 + \frac{3P_{el}}{c^2M_U/V}\right)}{r^2\left(1 + \frac{2GM_{U_0}r}{c^2R_er}\right)}$$
$$P'(r) = -\frac{\frac{GM_{U_0}}{R_e}r\left(1 - \frac{c^2M_U(r)/V}{c^2M_U(r)/V}\right)\left(1 - \frac{3c^2M_U(r)/(3V)}{c^2M_U/V}\right)}{r^2\left(1 - \frac{2GM_{U_0}r}{c^2R_er}\right)} = \frac{0}{0} \qquad (28)$$

The result is an indefinite quotient that is independent of r. This coincides with our assumption that at the edge the particles themselves should be in static equilibrium and it fits to the assumption that they do not move locally outside. If the edge of a newly formed particle moves away, the particles can move locally if they change the distance between the planes, i.e. the electrical pressure.

In addition, they can network more and more, which all together leads to the fact that the time of the particles moves in finite processes. The further away the edge of the universe is the more time passes. Or expressed in terms of processes, more processes can take place in the same unit of time. Time, as we understand it, is related to the interconnection to other particles, which in turn causes a permanent movement of individual particles in space and an inertia of the masses. Because the interconnections leads to a delay of the movement, since it takes time before all connections have experienced the new basic movement.

The indefinite term p'(r)=0/0 also changes the direction of the imagination. We no longer move in a resting space-time system with finite processes, but we come from the infinity of a resting time and spacelessness and develop slowly, through the resulting connections, towards the finiteness of time and the perceptibility of size and distance. This networking stands further as a piece of space and timelessness, only because everything is exchanged at the speed of light. I.e. it is timeless from the point of view of the exchange particles. The sum of all individual processes results in a closed, time and spaceless whole.

As long as everything moves and is connected to each other we experience finiteness, but the further we get to the edge of the universe, the less freedom we have and at the edge itself the inner movements seem to be dissolved, as if they were not there. Here outside one knows nothing about life deep inside.

In the second approach to solving Einstein's field equations, we now want to allow space to be a basic quantity that can stretch or compress. Also here we use a spherical symmetric approach, whereby our position should not play a role in it at first. So we look at the universe sphere from the outside and consider how the movements are when the masses increase at the edge.

If we take a three-dimensional hyper surface that can move in four-dimensional space and write the line element in spherical coordinates, then the Friedmann-Robertson-Walker metric looks like this

$$(ds) = (dt)^2 - r^2(t) \left[\frac{(dr)^2}{1 - kr^2} + r^2(d\theta)^2 + r^2 \sin^2\theta (d\phi)^2 \right] \quad (29) .$$

As mentioned above, R(t) is a scale factor, a scalar quantity that depends only on time and scales both the hyperspace and the coordinate system. If R(t) increases, it can mean that the spatial structure becomes larger undirected. Two points on it would therefore remove with time. But it does not have to be the space, it can also be the scalar time, we can distance ourselves also temporally, if the times run differently in the time systems. Here the approach with R(t) as space size is used, but this is not clearly defined. One can also take T(t) = R(t) instead of R(t) and would have a time lapse, which is removed with time.

k determines the curvature in equation (29). For k=0 we have a flat geometry, for k=1 the space is closed and for k=-1 open hyperbolic.

Since we again consider a spherically symmetric system, a large part of the components equals zero. The non-disappearing

Ricci tensor components are $R_{00} = -3\frac{\ddot{R}}{R}$ (30); $R_{ii} = \frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2} + \frac{2k}{R^2}$

(31) and the Ricci scalar $R = -6\left(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right)$ (32). If we put this

into the Einstein's field equations (1) together with the energy pulse tensor, we get

$$\frac{\dot{R}^2}{R^2} + \frac{kc^2}{R^2} = -\frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3} \qquad (33)$$

and the pressure equation

$$2\frac{\ddot{R}}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{kc^2}{R^2} = -\frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3} \quad (34) \ .$$

If the two equations are subtracted from each other, the following follows

$$\frac{\ddot{R}}{R^2} = -\frac{8\pi G}{3c^2}(\rho c^2 + 3p) + \frac{\Lambda c^2}{3} \quad (35)$$

Let us take equation (35) and think of the universe as unaccelerated $\ddot{R}=0$. Again, we use the approach that the pressure the planes exert on each other cancels the density

term as $p = -\frac{1}{3}\rho c^2$, then it follows that the constant Λ disappears. If we go with it into equation (34) and assume a flat curvature with k=0, then we get $\frac{\dot{R}^2}{R^2} = -\frac{8\pi G}{3}\rho$ (36) or transformed

$$\frac{\dot{R}^2}{R^2} = \frac{2\frac{4\pi G}{3}M_{U_0}R_U}{\frac{4\pi}{3}R_U^3R_e} = \frac{c^2}{R_U^3} \quad (37)$$

so
$$\frac{R}{R} = \frac{c}{R_U} = \frac{1}{T_U}$$
 (38).

Then the Hubble constant $H_0 = \frac{\dot{R}}{R}$, which is regarded as the movement of the scaling space size in relation to the distance, would be constant and equal to the age of the universe, i.e. a time size - the maximum time size at the edge of the universe. $\dot{R}(t)$ is interpreted as a velocity, a movement of space of the hyper plane itself. But this can also be rewritten into a time derivative $\dot{R} = \dot{T} \cdot c$ (39) and since it is only a scalar quantity we would have with $\frac{\dot{T} \cdot c}{R} = \frac{\dot{T}}{T} = \frac{1}{T_U}$ (40).

Then we have a solution that time moves at different distances at different speeds. The older objects are the further back they are in their time, because the time passes with increasing distance ever more slowly. We would then have a static universe, linearly enlarging at the edge, in which time runs at different speeds on the different spheres. Far away the time arrives not only late because the transmission is finite, but what we then see are also time processes, which run there slower than with us.

Two successive wave crests of the emitted distant light t_1 and $t_1 + \delta t_1$, as well as at the time when it arrives with us t_0 and $t_0 + \delta t_0$, have the same distance d(t) between transmitter and receiver.

$$d(t) = R(t) \int_{t_1}^{t_0} \frac{dt}{R(t)} = R(t) \int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{R(t)}$$
(41)

Or with T(t) and after the change of borders

$$d(t) = T(t)c \int_{t_1}^{t_0} \frac{dt}{T(t)} = T(t)c \int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{T(t)} \quad (42) .$$

If the scale factor changes only slowly during light propagation, this results in $\frac{\delta t_1}{T(t_1)} = \frac{\delta t_0}{T(t_0)}$ (43) or the wavelength

ratio of $\frac{\lambda_0}{\lambda_1} = \frac{T(t_0)}{T(t_1)}$ (44) and, by definition, the redshift z to

$$z + 1 = \frac{\lambda_0}{\lambda_1} = \frac{T(t_0)}{T(t_1)} \qquad (45)$$

According to Taylor applies to the scale factor developed by $\ensuremath{t_0}$

$$\frac{T(t_0)}{T(t_1)} = 1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2(t - t_0)^2 + \dots \quad (46) \quad \text{with} \quad H_0 = \frac{\dot{T}(t_0)}{T(t_0)} = const.$$

Thus follows for the redshift $z \approx H_0 ct$.

The further away the galaxies are, the more red shifted they become, because the time there is slower proportional to the distance.

According to (28), two worlds far apart are indeterminate in their basic approach as far as the total dynamic pressure is concerned, but the indeterminacy is not the same in the respective regions. And just as in the indeterminacy of time, movements of matter can develop, so these mechanisms run differently fast, depending on whether \dot{T} is large or small.

These time processes can also only be localized to the particles. Time itself can remain an abstract quantity, which is reflected in the processes, just like space no longer must have an independent existence. If light quanta arrive at us from a great distance, then they were not available to us in the meantime. For a quantum, the beginning and the end are simultaneous and there is no space in between, so it has not aged and you will not find the smallest structure of a piece of space. For us there are endless processes in the meantime, which have nothing to do with this quantum, the world continues without this quantum. Only when it arrives is it back in our worldly network and certain energy and impulse of the system with.