6. Entangled spin states

Christian Hermenau

The quantum-mechanical phenomenon of entanglement may show that the connections between the particles are not yet correctly interpreted. On the one hand it is not clear why elementary particles have a spin at all and on the other hand why a granular spatial structure cannot be detected even in extremely distant gamma-ray bursts.

At first, only the known derivations of entanglement shall be shown, and then further considerations on spin and space itself shall be made.

$\mathscr{H}^1$ and $\mathscr{H}^2$ are two independent Hilbert spaces with the base vectors $|\Psi^1_m\rangle$ and $|\Psi^2_n\rangle$. For this the direct product is explained for all vector pairs $|\Psi^1_m\rangle \otimes |\Psi^2_n\rangle$, or briefly $|\Psi^1_m\rangle|\Psi^2_n\rangle$. To each of the pairs a vector of a Hilbert space $\mathscr{H}$ shall be assigned, which is completely spanned by the vectors.

$$|\Psi^1_m \Psi^2_n\rangle = |\Psi^1_m\rangle|\Psi^2_n\rangle = |\Psi^2_n\rangle|\Psi^1_m\rangle$$

$|\Psi^1_m \Psi^2_n\rangle$ thus forms a basis of the direct product space of two Hilbert spaces.

$\mathscr{H}^1 = \mathscr{H}^1 \otimes \mathscr{H}^1 - \mathscr{H}^1 \otimes \mathscr{H}^1$ then each vector $|\Psi^{12}\rangle \in \mathscr{H}$ can be represented as a linear combination of $|\Psi^{12}\rangle = \sum\limits_{m,n}\int c_m |\Psi^1\rangle|\Psi^2\rangle$ (1).

In addition, the direct product of the vectors

$$|\Psi^1\rangle = \sum_{m,n}\int a_m |\Psi^1{}_m\rangle \in \mathscr{H}_1$$

$|\Psi^2\rangle = \sum\limits_{m,n}\int b_n |\Psi^2{}_n\rangle \in \mathscr{H}_2$ (2) should be represented by

$$|\Psi^{12}\rangle = |\Psi^1\rangle|\Psi^2\rangle = \sum_{m,n}\int a_m b_n |\Psi^1{}_m\rangle|\Psi^2_n\rangle \in \mathscr{H}.$$

(The integral stands for continuous fractions)

In product room $\mathscr{H}$, a scalar product is explained which is used for $\left|\Psi^{12}\right\rangle = \sum_{mn}\int c_m \left|\Psi^1_m\right\rangle\left|\Psi^2_n\right\rangle$ and $\left|\Psi^{12}\right\rangle = \sum_{r,s}\int d_m \left|\Psi^1_r\right\rangle\left|\Psi^2_s\right\rangle$

is given by $\left\langle\chi^{12}\|\Psi^{12}\right\rangle = \sum_{m,n,r,s}\int c_m d^*_n \left\langle\Psi^1_r\|\Psi^1_m\right\rangle\left\langle\Psi^2_s\|\Psi^2_n\right\rangle = \sum_{m,n}\int c_m d^*_m$ (3).

Now applies to separable elements $\left|\Psi^{12}\right\rangle = \left|\Psi^1\right\rangle\left|\Psi^2\right\rangle$ and $\left|\chi^{12}\right\rangle = \left|\chi^1\right\rangle\left|\chi^2\right\rangle$ after (2) $\left|\chi^1\right\rangle\sum_r\int f_r\left|\Psi^1{}_r\right\rangle$ and $\left|\chi^2\right\rangle\sum_r\int g_s\left|\Psi^2{}_s\right\rangle$ it follows

$$\left\langle\chi^{12}\middle|\Psi^{12}\right\rangle = \left\langle\chi^1\middle|\Psi^1\right\rangle\left\langle\chi^2\middle|\Psi^2\right\rangle$$

then $c_{mn} = a_m b_n$ and

$d_n = f_r g_s \left\langle\chi^1\middle|\Psi^1\right\rangle\sum_{m,r}\int a_m f^*_r\left\langle\Psi^1_r\middle|\Psi^1_m\right\rangle = \sum_m \int a_m f^*_r \left\langle\chi^2\middle|\Psi^2\right\rangle = \sum_n\int b_n g^*_n$ and

$\left\langle\chi^1\middle|\Psi^2\right\rangle = \sum_{mn}\int b_n g^*_n$ with (3) follows

$\left\langle\chi^{12}\middle|\Psi^{12}\right\rangle = \sum_{mn}\int a_m b_n f^*_m g^*_n = \sum_m\int a_m f^*_m \sum_m\int b_n gs^*_n$.

However, the states of (1) also include those, that are not separable as a product and are described as an entangled state. For separation, the coefficients c_{mn} of the products would have to be decomposable into $c_{mn}=a_m b_n$. From the linear dependence of the rows of the matrix $M_{mn}:=a_m b_n$ follows $\det(c_{mn})=0$ and the c_{mn} can be chosen so that this condition is not fulfilled.

The basis of the states of spin-1/2-particles is formed as a product of location and spin states $\Psi(x,s) = \Psi(x)u(s)$ with $u(s) = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$.

Two-particle states are then

$$\Psi_1(x_1,s_1)\Psi_2(x_2,s_2) = \Psi_1(x_1)\Psi_2(x_2)u_1(s_1)u_2(s_2) .$$

The general state is a superposition of such product states.

As basis of the one-particle states we take the eigenstates

$u_+ = \begin{pmatrix}1\\ 0\end{pmatrix}$ and $u_- = \begin{pmatrix}0\\ 1\end{pmatrix}$.

A basis of the two-particle spin states is then formed by the states

$u_{1+}u_{2+}$, $u_{1+}u_{2-}$, $u_{1-}u_{2+}$, $u_{1-}u_{2-}$.

The most general spin state can be superposed from the basic states. The spin state σ of the entire system is defined by $\sigma=\sigma_1+\sigma_2$ except for the factor $h/2$.

For the symmetric and the antisymmetric superposition the following applies

$$\Psi_s = \frac{1}{\sqrt{2}}\left[u_{1+}u_{2-} + u_{1-}u_{2+}\right]; \qquad \Psi_s = \frac{1}{\sqrt{2}}\left[u_{1+}u_{2-} + u_{1-}u_{2+}\right] \quad (4).$$

Both are a common eigenstate of the two operators σ^2 and σ_z.

It applies $\sigma^2\psi_a=0$, $\sigma_z\psi_a=0$, $\sigma^2\psi_s=8\psi_s$, $\sigma_z\psi_s=0$.

$\sigma^2\psi_s=8\psi_s$ means with a total spin of s=1 for ψ_a results in s=0 and the two states ψ_s and ψ_a are not factorable so entangled. They can thus be described as the product of a one-particle state.

From (4) results $u_{1+}u_{2-} = \frac{1}{\sqrt{2}}(\Psi_s + \Psi_a)$, $u_{1-}u_{2+} = \frac{1}{\sqrt{2}}(\Psi_s - \Psi_a)$ and thus

$$\sigma^2 u_{1+}u_{2-} = \frac{1}{\sqrt{2}}(\sigma^2\Psi_s + \sigma^2\Psi_s) = \frac{8}{\sqrt{2}}\Psi_s = \sigma^2 u_{1-}u_{2+} \quad (5)$$

A measurement from σ^2 means that the eigenvalues ψ_s and ψ_a are superposed to the eigenvalues 8 and 0. Since also their amplitudes are equally large their eigenvalues or the total spin 1 or 0 must be found with the same 50% probability.

This means now that if we measure the spin component σ_{1z} at a particle 1, which moves to the left, then one finds to 50% probability the eigenvalue +1. After the measurement the system is in the state $u_{1+}u_{2-}$ with the property $\sigma^2 u_{1+}u_{2-} = u_{1+}u_{2-}$.

Moreover $\sigma_{2s}u_{1+}u_{2-} = -u_{1+}u_{2-}$, the second particle has the eigenvalue -1.

In the remaining 50% of the cases the 1st particle has the eigenvalue -1 and the 2nd particle the eigenvalue +1.

This is independent of their distances.

According to the Copenhagen interpretation, it makes no sense to determine any property of a particle as long as no measurement has been made on it. The two particles are regarded as an inseparable unit $\Psi_s = \left[u_{1+}u_{2-} - u_{1-}u_{2+}\right]/\sqrt{2}$ (5) as long as no measurement is made. At the moment of the measurement on the 1st particle there is an instantaneous effect on the 2nd particle independent of the distance.

This remarkable result does not contradict the theory of relativity, not because the determinism could be violated, but because this is exactly the actual meaning of an information transfer at the speed of light. A quantum that moves with c in our matter-space-time image must, according to our ideas, have a position at a certain point in time. This is the view from our resting system. For the quantum, however, time stands still and space disappears in the direction of motion. Beginning and end are instantaneous. Each measurement means a determination of the quantum back into our space-time. A quantum in the middle of its movement cannot be measured or fixed because there is no in-between for the quantum.

When the spin 1/2 quantum is entangled exactly this property of all particles moving with c becomes clear. There is a countable spatial size for us, which is however determined by our course of time. The space then seems to be granular, but of such a small structure that it is almost indifferent again. Nevertheless, very precise experiments over extremely large distances have shown that no dispersion can be detected, which would have been to be expected for gamma-ray bursts over extremely large distances with a grain size in the range of the Planck lengths. Space does not have to have an independent structure; it can only have reality for our resting world.

Most of the connections between the particles run through bosons and thus in a temporary timeless world. Even entangled particles are indeterminate in their quantum properties as long as they are not measured. Only through measurement do they reconnect with our world and simultaneously detach themselves from the 2nd partner particle.

The idea now is that the entanglement is not completely erased, but that the superimposed state is reduced to a respective tiny contact time. In an electrical or gravitational exchange, the two particles briefly exist together in a superimposed one-particle state that is independent of the distance. Beginning and end are for a short moment $t_0 = 1\cdot 10^{-23}$s spatially and temporally indeterminate. Nevertheless, the centre of gravity remains essentially the same at the same location except for a small blur, since the inert position is isotropic distributed to other particles on statistical average.

This does not necessarily apply to spin. In the following it shall be shown that the intrinsic spin of the electron or the proton can be explained by a continuous entanglement of both particles.

In an inhomogeneous magnetic field, as shown in the Stern-Gerlach experiment, an atomic beam is deflected by its magnetic moment μ with the force $F = \nabla(\mu\cdot B)$ (6). In quantum mechanics, the corresponding operator equation looks like this: $\hat{\mu} = \left(\frac{q}{2m}\right)\hat{L}$ (7), $\hat{L}$ is the angular momentum operator of the electron on its orbit.

The eigenvalues μ^2 provide the square of the magnetic moment. With $\hat{L}\psi = \hbar l(l+1)\Psi$ (8) one obtains $\hat{\mu}^2\psi = \left(\frac{q}{2m}\right)^2 \hat{L}^2\psi = \left(\frac{q\hbar}{2m}\right)^2 l(l+1) = \mu^2\Psi$ (9) it then follows $\mu^2 = \left(\frac{q\hbar}{2m}\right)^2 l(l+1)$ (10). Now this is also true in the ground state of hydrogen at l=0 and μ=0, but each electron nevertheless has an intrinsic angular momentum with the magnetic moment μ_s which is called spin. This spin and its associated spin operator $\hat{S}$ have the same properties as an angular momentum operator. Its eigenvalues for the component s_Z have the values $\hbar m_s$ with m_s= -s, -s+1 …., s-1, s and the value s=1/2.

The following properties apply to the electron spin. The spin operator is hermitic and fulfills the rules of exchange $[S_i, S_k] = i\hbar S_l$; i, k, l, are cyclically. The spin is the value 1/2 and the spin space is defined by exactly two linearly independent states

$|s\rangle \quad (|\uparrow\rangle; |\downarrow\rangle)$. The complete condition is described by vectors from the product space $\mathcal{H} = \mathcal{H}^x \otimes \mathcal{H}^S$. A magnetic moment μ is connected to S and the Hamilton equation applies to the movement with spin

$$H = H_0 - \frac{g_s q}{2m} B \cdot S \quad (11).$$

Thus the general quantum-mechanical equation of motion in the Schrödinger image with spin particles is

$$i\hbar\partial_t |\Psi\rangle = H_0 |\Psi\rangle - \frac{g_s q}{2m} B \cdot S |\Psi\rangle \quad (12)$$

The components to each $|\Psi\rangle$ state are the Pauli spinors $\Psi_+(x,t) = \langle x,1/2|\Psi\rangle \quad \Psi_-(x,t) = \langle x,-1/2|\Psi\rangle$ (13) and the corresponding adjunct spinors $\Psi_+^*(x,t)$; $\Psi_-^*(x,t)$.

$\hat{S}$ behaves like an angular momentum operator. The movement of the electrical charge of the electron associated with rotation generates the magnetic moment of the electron itself. This would mean, however, that the electron has a finite expansion, which is not necessarily assigned to the electron. A peculiarity of the interpretation of spin as rotation of the electron itself is that the eigenvalues of S_z have only the values $s = \pm\hbar/2$. Moreover, it is not clear why the electrons rotate at all. Since one does not necessarily want to give the electron a size and the spin has exactly half a angular momentum as eigenvalue, one is reluctant to compare the spin completely with an angular momentum and generally understands it as an own quantum mechanical state.

Let us nevertheless assume that the electron, according to our picture, has a structure of two planes of size R_e^2 which rotates classically with $R_e/2$ around its centre of gravity with its mass M_e and an angular velocity of $\omega_0 = 2\pi f_0$ ($f_0=1/t_0$), then for the classical angular momentum follows $L_z \approx 2.6 \cdot 10^{-36} Js$ which is about 80 times too small a value to be associated with our rotation in space.

In our picture, the electron and the proton clearly assume three different cyclic spatial positions and corresponding connections. In our space-time image a mass angular momentum should result from this. The spin then stands for a motion which, compared to a translational motion, rotates at the speed of light or a sequential temporal sequence which takes place with our minimum time pulses t_0. This means that the spin rotation must not be inert, which would be conceivable with a sequence of connections. In this picture, the planes themselves are not regarded as the seat of mass, but only the distance between the planes says something about mass, as a reference value. The movements of the planes in space, in contact with other particles, are delayed; here the change of the movement depends on the respective distance to the particles. The rotation around the centre of gravity itself only has meaning for a single opponent, with whom it has contact, and this can be interpreted as a short-term entanglement, as well as a classical rotation of two non-separated particles, the electron and the associated proton.

From this point of view it makes sense that the spin cannot be stopped or consumed but always has the same fixed value. It results from the changing short-term connections to a whole and does not have the same meaning as the angular momentum operator $\hat{L}$, which moves and is changeable in our space.

Perhaps the quantum state of the spin remains entangled and we conclude only indirectly via its magnetic moment μ_s on its size and its direction and infer from this the spin size of $h/2$. But if we see the rotation of the electron and the proton further than superimposed, then the spin 1/2 state is omitted and related to a radius of $R_e/2$ we could assign the rotation a classical angular momentum for both masses geometrically averaged ($\sqrt{m_e m_p}$), which then leads to an exact result $h/2$ given at our fixed exchange time t_0.

Then both, the proton and the electron have the same spin, which presents itself to us as $h/2$ a measurement. Then the particles with t_0 exchange themselves permanently, they change between inertial indefinite and electrically determined connections. From the point of view of the exchange particles, these connections are timeless and from our point of view they show up in their magnetic moment. If we measure the moment, we force the electron to separate from the proton for the measurement. This gives us a value that is positioned on our world and statistically halved.

The extremely precise charge neutrality of electron and proton can also make sense with the help of entanglement. We give the electron a position on an inner stable position around a proton, which, due to the many connections to other particles (gravitational), no longer lies in the nucleus but in the range of uncertainty h. And yet the charges in the neutral atom are related to each other and cannot be measured externally. An electrically uncharged atom that is not measured is like a common superimposed state where the exact position is ambiguous or non-existent. The charges do not behave like two spatially separated bodies, but like a whole, with only one centre of mass position which is without charge size. So charges could also be any small in size. If one tries to estimate the size of a charge with a charge- entangled neutron in a measurement, the electron will generally not form a connection with the neutron and the cross sections can be correspondingly very small.

If entangled states between particles are actually time- and space-less, this can only be realized geometrically with particles as planes. With point-shaped particles we would have no direction vector and we would need an independent space-time structure, which is open again just by the entanglement. But also spherical particles don't make sense anymore because of the entanglement, because we would have to explain a short phase of a space curvature at the beginning and at the end. How can the quanta change from a convex initial curvature at the start particle to a concave final curvature at the end particle? Exactly these difficulties would not occur with fixed plane structures. A measurement of the quanta from our world view would then result in the expected plane waves.