1. Is our Universe closed?

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A Consideration about a structure of the universe, which as a lower boundary should have the Schwarzschild radius as an insurmountable size and that as an upper limit should have the radius of the universe. The idea is that we are in a linearly growing universe, where matter is newly added at the outer edge, and where its elementary mass size belongs to the respective spherical shell.

According to quantum field theory, the vacuum does not consist of a emptiness, but has an energy value even in the complete absence of fields and particles. This can be e.g. zero point fluctuations or particle-antiparticle-pairs, which originate in very short terms and decay again. The theoretically predicted energies which are in such a vacuum are extremely large and exceed the measured values e.g. with the Casimir Effect by many orders of magnitude (factor 10^{120}). They are also as a candidate for the dark energy to many orders of magnitude too large.

Nevertheless, the idea of a non-empty vacuum should also be the basis for the formation of universes in this approach. But the idea should be continued further. The basis should be an indefinite, blurred form of space and time, which has an undefined number of dimensions. The quantities, outside the universe, such as energy and momentum cannot be localized as particles there, which lead to the fact that there are no networked or complex forms, but also the terms such as very large or very small, long or short, lose their meaning. The forms would be more comparable with numerical values, which are irrelevant and can be deleted as a whole at any time. They can be present at any place in any number at any time and yet have no meaning for a non-existent whole.

The significance of a fixed permanent quantity in a fixed space, in which the time processes run in chronological order, in which the structures are sharp, can exchange, connect and grow is only possible within a closed system, with a countable number of limited elements, for a finite time. The meaningful, important, real then exists only for the elements in a closed system. Information about it cannot reach the outside world and thus get a status of eternity, because this outside world does not allow it.

The whole system does not have to aspire from one point into this vacuum, but it can also develop and allow a particle of fixed size to become real for us. Any other existing value does not get access to the system and remains irrelevant, no matter how its energy, space or time values are; they do not fit into the whole.

A universe then functions like a filter which, because of certain initial parameters, only allows a single structure of infinitely many possible ones.

Some basic quantities in our universe are the three-dimensional space, the time, the finite speed of propagation, the stable mass of the proton and that of the electron.

According to the standard model, the world was created in a kind of Big Bang in an extremely small space. Within a tiny time lapse the space, the time, the energies and then the four interaction forces formed, one after the other. According to the theory of inflation, the universe was dramatically inflated before it became physically comprehensible for our understanding. Matter condensed from energy and even some helium nuclei were fused during the universe cooled down.

This beginning does not contradict the possibility that gigantic energies are in the vacuum, it has however the disadvantage that once developed sharp structures, such as protons and electrons, do not disappear any longer. Objects are formed from a meaningless initial state, which have the potential to store information of meaning at least formally to last forever.

One of the basic foundations of Big Bang Theory is the redshift of the galaxies. The further away, the more red shifted the spectral lines are, which suggest that the galaxies are striving apart. On the other hand we have the background radiation, which is interpreted as the remnants of the Big Bang and can be remarkably integrated into the theory. The numerical proportions of protons, helium nuclei and higher metals also fit this theory.

But what causes great explanatory difficulties is the question of the symmetry of matter and antimatter and the concept of matter itself. Why is the proton heavy and the electron light? What is the dark matter, without which the Big Bang theory would not work, without which large systems could not have formed so quickly?

The theoretical approach of Big Bang theory is not the only solution to Einstein's equations and is rather exotic among the world models, but it is the only model that is so well compatible with the observations found.

Nevertheless, serious conditions such as inflation, dark matter and symmetry deviations were introduced in order to make them suitable in detail.

Without additional unknown parameters one will probably not find a solution approach of Einstein´s equations, which is compatible with reality, but just as well it can be also other special unknowns, which continue a theory, but perhaps without inflation or dark matter and dark energy.

Besides this neither to the charge nor to the gravity a holistic beginning is found and there is also no reason why matter is to be localized only blurred, it must be assumed as given.

The considerations here should focus on that the system is self-enclosed, the finiteness and the precisely defined number of objects that should dissolve again sometime. The beginning must be precisely defined and only gradually mix, then network and thus become increasingly complex. Nevertheless, this should be limited in time. At some point, therefore, the processes must dissolve piece by piece, simplify, and then disappear again into the blur. Such an approach does not fit an explosive development, a sudden presence, but rather a continuous growth process, from the simple to the increasingly complicated. With these preconditions we then have a very first particle, which determines with its space size, with its volume area and mass as well as the step sizes, the further mass sizes.

This first particle only has meaning for our future world, because it is the first one and it is of the same kind as we are. This particle can either disappear immediately, or it can become larger, it can grow. However, it must observe fixed laws that cancel each other out as a whole. This first particle should have the size of the electron radius and be comparable with a neutron, which however has a much higher mass equivalent. Then a temporally finite process can begin, so that a three-dimensional space becomes possible, which increases with a step length of always one electron radius R_e and has thereby the border velocity c fixed with it. Thus the universe grows spatially linear, the radius of the universe is $R_U = k \cdot R_e$ with k = 1, 2, 3, 4 ... and the speed is with $c = R_U / t_U$ the speed of light.

How does this developing volume remain closed? Why shouldn't any particle of the vacuum within the volume change the inside?

Self-contained systems to which we have no access to its interior are for example black holes. According to Einstein's equations, there is a theoretical possibility that matter collapses in such a way that nothing escapes from a fixed radius, which depends only on the total mass.

Let us first investigate the Schwarzschild metric inside and outside black holes. Let's start with the known solutions of static equilibrium conditions for stars with constant density.

Starting with Einstein's field equations

$$R_{ij} = \frac{8\pi G}{c^4}\left(T_{ij} - \frac{1}{2} g_{ij} T\right) \quad (1),$$

we can take a time-independent, spherically symmetric approach with $\nu = \nu(r)$ and $\lambda = \lambda(r)$ the general approach

$$ds^2 = e^{\nu(r)} c^2 dt^2 - e^{\lambda(r)} dr^2 - r^2 (d\vartheta^2 + \sin^2\vartheta d\varphi) \quad (2).$$

Pressure and density should be isotropic and also spherically symmetrical, then p = p(r) and $\rho = \rho(r)$ for the energy density $U(\lambda) = \left\{ e^{\nu/2} c, 0, 0, 0 \right\}$ and after some transformations for the energy impulse tensor $T_{00} = e^{\nu} \rho c^2$ and accordingly for $\left\{ T_{11}, T_{22}, T_{33} \right\} = \rho \left\{ e^{\lambda}, r^2, r^2 \sin^2\vartheta \right\}$ follows $e^{\lambda} = (1 - \frac{gM(r)}{c^2 r})^{-1}$ and $e^{\nu(r)} = 1 - \frac{2GM}{c^2 r}$ which leads us to the Oppenheimer-Tolman-Volkoff equation

$$p'(r) = \frac{GM\rho[1 + p(c^2\rho)][1 + 4\pi r^3 / (c^2 M)]}{r^2 [1 - 2GM / (c^2 r)]} \quad (3).$$

With the corresponding equations $M'(r) = 4\pi r^2(r)$, $p/p_0 = (\rho/\rho_0)^\gamma$ and the boundary conditions M(0)=0, P(R)=0.

With the idealized form of a constant density ρ_0 within the radius and a mass to which applies $M(r/R)^3$, one finds the solution:

$$p(r) = \rho_0 c^2 \frac{\sqrt{1 - r_s r^2/R^3} - \sqrt{1 - r_s/R}}{\sqrt[3]{1 - r_S/R} - \sqrt{1 - r_S r^2/R^3}} \quad (4), \quad \text{with } r_S = \frac{8\pi G \rho_0 R^3}{3c^2}$$

as the Schwarzschild radius.

The maximum pressure is highest in the center because of $P'(r) = 0$ with $M(r)/r^2 \approx r = 0$ and has the maximum value

$$p(0) = \rho_0 c^2 \frac{1 - \sqrt{1 - r_S/R}}{\sqrt[3]{1 - r_s/R} - 1}. \quad (5)$$

This equation has only stable equilibrium solutions for $R > 9r_s/8$, otherwise diverged (5). With r_s the equilibrium condition then is: $\frac{GM}{c^2 R} < \frac{9}{4}$ (6)

This means that there is a maximum mass and a minimum radius at which a star is still stable.

In a much more complicated form, it can be shown that (6) is a general limit independent of specific assumptions. This again does not mean that matter can actually become unstable and assumes the state of a black hole, where matter disappears in a singularity.

The last known equilibrium configuration is the neutron star. Here we look at a neutron star, with the mass of our sun, which has shrunk to about 16 km radius. This corresponds to the matter density of heavy atoms, which is for iron $m_{Fe} = 9.4 \cdot 10^{-26} kg$ with a core radius of $R = 5.6 \cdot 10^{-15} m$ at an average density of $\rho \approx 10^{17} kg/m^3$. The electron gas is relativistic degenerate and is mainly pressed into the protons, because they do not have enough space in the phase space.

In the outer regions of the star, the density initially increases from $\rho = 10^7 kg/m^3$ to $\rho = 4 \cdot 10^{14} kg/m^3$ in 1 km depth. Then the nuclei gradually begin to dissolve and we find an ever higher neutron number. From a density of around $2 \cdot 10^{17} kg/m^3$, the neutrons show properties of suprafluids, but in these areas the equations of state are only insufficiently known. There is also an upper limit for the neutron stars, which is after (6) $1{,}8 \cdot M_o$.

According to (6) it is therefore very conceivable that a neutron star increases even more with mass and then exceeds any pressure, no matter how high. If, however, we then give up the neutron spacing and the neutrons are even further pushed into each other, then we get into areas where the charges are no longer neutral and also the elementary single charges become larger than one e.

According to (5), however, even such high charge forces would no longer be able to stop the divergent increase in pressure. The matter would then fall into a black hole and disappear into a singularity.

Now in this universe other than whole elementary charges are not known, nor has any known structure a size, which is much smaller than the atomic nucleus. It is also not known how far the theory of relativity still applies in quantum mechanical domains. What if the smallest charge sizes are an elementary charge, which have a size of about $10^{-15} m$ and cannot be compressed any further? What, if the system as a whole does not allow this?

One possibility would be that the central region of a neutron star is much larger. That the Schwarzschild radius is gaining importance much earlier, which previously with a coordinate transformation was fixed. If the Schwarzschild radius is not exceeded with large mass accumulations inside, then there must be an inner radius R_i, which is always larger than the Schwarzschild radius. The pressure would then already become maximal, if this inner radius only approaches the Schwarzschild radius more and more, without it ever being able to reach it. Even then the pressure increases with increasing mass, up to the neutron liquid and even further, but the neutrons would already come earlier into the range, at which the electrical charges begin again.

Let us simply assume that, in a neutron star, we are no longer dealing with spherical charges of the size R_e, but with two charge planes which face each other like two plates and have a total area of that of an electron. Then, applies after the Gaussian theorem

$$F = \int_V \lambda E d\tau = \int_V E div E d\tau = \varepsilon_0 \int_V nn \cdot \nabla E^2 / 2 d\tau + \varepsilon_0 \int_V n E^2 div n d\tau \quad (7)$$

The volume element is then $d\tau = dl df$. The first integral to the right of (7) disappears, because for l < f applies $dl \to 0$ and with a few calculations the second one becomes

$$F = \frac{\varepsilon_0}{2} \int_F E^2 n df \quad (8)$$

with which then results $F = \frac{\varepsilon_0}{2} \int_A E^2 n df = \frac{e^2}{2\varepsilon_0 A_e}$ (9),

which corresponds to a repulsive force of 16 N per charge carrier.

After (6), at a density of $\rho = 5 \cdot 10^{16} kg/m^3$ we get a central pressure of $p(0) = 1 \cdot 10^{29} Pa$. The density is somewhat lower because here we do not use iron but neutrons and take the electron radius as the radius of the charges. It has not yet been taken into account that the center can only be reached up to 3 km, the Schwarzschild radius. Nevertheless, the force acting on a surface A_e at 7 N is smaller than the repulsive electric force.

If it goes strictly to (6), with $1.8 \cdot M_o$ any counter pressure would broke through. However, if the matter does not fall by $r = r_S$ into a black hole, but the inner radius r_S holds and grows with it, then the pressure at the Schwarzschild radius falls again with increasing mass, because the matter now in relation to the total radius R, becoming thinner and thinner spherical shell distributed. The ratio r_S / R then becomes smaller and smaller, but (4) can no longer be applied because the mass distribution no longer runs homogeneously to the center, but is distributed in a layer around a large void. Nevertheless, it can be said that the compressive forces decrease with increasing mass and radius, and the equilibrium position stabilizes further.

Another reason why masses would not disappear in a black hole would be that weak counter-accelerations, which arise later with our approach, become very strong at large mass concentrations and thus prevent a disappearance behind an event horizon, so that the entire system remains closed and reversible.

We neither reach the state of an ideal black hole, nor does matter disappear behind an event horizon and we do not need to worry about possible singularities.

Thus, the Schwarzschild radius would represent a lower limit, shielding the interior, from which neither anything comes out, nor something falls in, and yet it is of finite spatial size.

What about the other direction in the macrocosm? What if we look at the universe as a whole, is there also a boundary that keeps the cosmos closed and later reverses the processes?

With these considerations the space is supposed to increase with the speed of light, i.e. the cosmos is at 13.7 billion years also only approximately $10^{26} m$ large. It is apparently isotropic and homogeneous on large scales and its mass lies according to the latest findings, together with the dark energy and dark matter at about $10^{52} kg$. Thus we would have coincidentally exactly an average density of the ratio size one, for an eternally continuously expanding universe.

Although the universe is supposed to be isotropic on a large scale, we take Schwarzschild's spherical symmetric solution approach and pretend to approach this universe from a void at the outside $T_i^j = 0$. Then we notice that the total mass at this radius could correspond exactly to the Schwarzschild condition. Our world would then be the inside of a black hole.

So let us look at the solutions of Einstein's equations for the inside of black holes. We then have to deal with time-dependent solutions, because without additional conditions no stable equilibrium configurations are possible. The masses should show sphere-symmetry and a matter-pressure is negligible.

Starting from (2) we go to co-moved coordinates. ϑ and φ remain meaningfully the same. r´ and t´ should be designed so that they are constant in case of a free fall and the time is equal to the Eigen-time. It should apply $r,t \to r',t'$

$dr = (\partial r / \partial t')dt' + (\partial r / \partial r')dr'$ and $dt = (\partial t / \partial t')dt' + (\partial t / \partial r')dr'$

The line element is then given in the form

$$ds^2 = A(t',r')c^2dt'^2 + B(t',r')cdt'dr' + C(t',r')dr'^2 + r^2(t',r')(d\vartheta^2 + \sin^2 d\varphi^2) \ . \qquad (10)$$

In radial direction applies $dr' = 0$ and it applies $d\varphi = 0; d\vartheta = 0$. For the Eigen-time follows $dt' = ds/c = \sqrt{A}dt'$, so must be $A(r',t') \equiv 1$. With the new coordinates applies with the Eigen-time t´

$d(\partial F/\partial t')/dt' - \partial F/\partial r' = 0$.

Let's set for F: $F = (ds/dt')^2 = c^2 + Bc\dot{r}(t') + C\dot{r}^2(t')$

Then follows $\frac{d}{dt'}(Bc + 2C\dot{r})^2 = c\frac{\partial B}{\partial t'} + c\frac{\partial B}{\partial r'}\dot{r} + c\frac{\partial C}{\partial t'}\dot{r}^2 + 2\ddot{r} = c\frac{\partial B}{\partial r'}r' + \frac{\partial C}{\partial r'}\dot{r}$ (11)

For $r'(t') = const$ a solution to be this equation, $\partial B/\partial t' = 0$ or B=B(r´) must apply.

With $A \equiv 1$, follows then for the line element with the renaming $r' \to \varsigma$ to

$$ds^2 = c^2 d\varsigma^2 - U(\varsigma,\tau)d\rho^2 - V(\varsigma,\tau)(d\vartheta^2 + \sin\vartheta d\varphi^2) \qquad (12)$$

We also assume an idealized constant spatial density:

$$\rho = \begin{cases} \rho(0) & \text{for } 0 \le \rho < R \\ 0 & \rho > R \end{cases} \qquad (13)$$

The gas is then determined only by gravity, which is assumed to be free falling.

Thus the spatial elements disappear and we get only $U_\mu = \{c,0,0,0\}$
So the impulse-energy-tensor has only the component T_{00}

$$T = g^{\lambda\mu}T_{\lambda\mu} = g^{00}T_{00} = T_{00} = \rho c^2 \quad \text{so with} \quad S_{\mu\nu} = T_{\mu\nu} - (g_{\mu\nu}/2)T$$

follows $\{S_{00}, S_{11}, S_{22}, S_{33}\} = \frac{\rho c^2}{2}\{1, U, V, V\sin^2\vartheta\}$ (14)

With the help of the Ricci tensor then follows for the field equations:

$$R_{00} = \frac{1}{c^2}\left[\frac{\partial_{\tau\tau}U}{2U} + \frac{\partial_{\tau\tau}V}{V} - \frac{(\partial_\tau V)^2}{2V^2}\right] = -\frac{4\pi G}{c^2}\rho \qquad (15.a)$$

$$R_{11}=\frac{1}{c^2}\left[\frac{\partial_{\tau\tau}V}{2}+\frac{(\partial_\tau U)^2}{4U}-\frac{(\partial_\tau V)(\partial_\tau V)}{2V^2}\right]+\frac{\partial_{\varsigma\varsigma}V}{V}-\frac{(\partial_\varsigma V)^2}{2V^2}-\frac{(\partial_\varsigma U)(\partial_\varsigma V)}{2UV}=-\frac{4\pi G}{c^2}\rho U \quad (15.b)$$

$$R_{22}=-1-\frac{1}{c^2}\left[-\frac{\partial_{\tau\tau}V}{2}-\frac{(\partial_\tau V)(\partial_\tau V)}{2V^2}\right]+\frac{\partial_{\varsigma\varsigma}V}{2U}-\frac{(\partial_\varsigma U)(\partial_\varsigma V)}{4U^2}=-\frac{4\pi G}{c^2}\rho V \quad (15.c)$$

$$R_{33}=\frac{1}{c}\left[\frac{\partial_{\tau\varphi}V}{V}-\frac{(\partial_\tau V)(\partial_\varphi V)}{2V^2}-\frac{(\partial_\tau U)(\partial_\varphi V)}{2UV}\right]=0 \quad (15.d)$$

15.a to 15.d are solved using the separation approach $U(\rho,\tau)=R^2(\tau)h(\varsigma)$, $V(\varsigma,\tau)=S^2(\tau)g(\rho)$ what with (15.d) leads to $S(\tau)=const\cdot R(\tau).$

The g and h should be chosen so that the constant is always 1, then follows $R(\tau)=S(\tau)$.

If we now go with $\varsigma'^2=g(\varsigma)$ to a new radial coordinate, then at first $h(\varsigma)d\varsigma^2=f(\varsigma')d(\varsigma')^2$ and with the renaming $\varsigma=\varsigma'$ we get then

$$U(\varsigma,\tau)=S^2(\tau)f(\varsigma') \text{ und } V(\varsigma,\tau)=S^2(\tau)\varsigma^2$$

With (15) the following equations remain

$$3\ddot{S}(\tau)S+4\pi G\rho S^2=0 \quad (17)$$

$$\ddot{S}(\tau)S+2\dot{S}^2(\tau)-4\pi G\rho S^2=-\frac{c^2 f'(\varsigma)}{\varsigma\cdot f^2} \quad (18)$$

$$\ddot{S}(\tau)S+2\dot{S}^2(\tau)-4\pi G\rho S^2=-\left[-\frac{1}{\varsigma^2}+\frac{1}{\varsigma^2 f}-\frac{f'(\varsigma)}{2\cdot\varsigma\cdot f^2}\right] \quad (19)$$

If S is eliminated from (18) and (19), the following is obtained for $f(\varsigma)$: $f'(\varsigma)=2f/(f-1)/\varsigma$ their solution

$$f(\xi)=\frac{1}{1-k\varsigma^2} \quad (20)$$

with the constant k.

Together with (17) follows

$$\frac{S\ddot{S}}{2}+\dot{S}^2-2\pi G\rho S^2+kc^2=2S\ddot{S}+\dot{S}^2+kc^2=0 \quad (21)$$

From (17) and (21) $\ddot{S}$ can be eliminated what leads to the equation

$$\dot{S}^2+kc^2=\frac{8\pi G\rho S^2}{3} \quad (22).$$

Multiply (22) by S and differentiate according to τ follows

$$d(\rho S^3)/d\tau=\frac{3\dot{S}}{8\pi G}(2S\ddot{S}+\dot{S}^2+kc^2)=0 \qquad => \qquad M_0=\frac{4\pi}{3}S^3\rho=const.$$

M_0 is the total mass within radius S, which is independent of the exact spatial distribution. With (22) then follows

$$\dot{S}^2-2GM_0/S=-kc^2 \quad (23)$$

With the initial condition $\dot{S}(0)=0$, i.e. the static beginning follows for the integration constant

$$k=\frac{2GM_0}{c^2S(0)} \quad (24)$$

and (23) becomes

$$\dot{S}^2=kc^2\frac{S(0)-S(\tau)}{S(\tau)} \quad (25)$$

The solution of (25) in implicit form is as follows

$$S(\tau)=\frac{S(0)}{2}(1+\cos\vartheta), \qquad c\tau=\frac{S(0)}{2\sqrt{k}}(\vartheta+\sin\vartheta) \quad (26)$$

Thus with (12) the inner metric of a black hole is

$$ds^2=c^2d\tau^2-S^2(\tau)\left[\frac{d\rho^2}{1-k\rho^2}+\rho^2\left(d\vartheta^2+\sin^2\vartheta d\varphi\right)\right] \quad (27)$$

(27) is now a Robertson-Walker Metric, as it is also known from cosmology for the universe as a whole. The initial conditions are set so that the system was at rest at the beginning and the total mass of radius S is spatially constant at a fixed point in time. Let us consider only a spherical shell of the size $R_U=S(\tau)$ equal to the Schwarzschild radius

and our universe time. Then we would have used the same metric that is used to describe the universe, with the condition that, for a reason yet to be clarified, the masses do not move towards the center.

This means that the idea that we are inside a black hole does not contradict the observations under these hypothetical conditions.

The crucial question remains, do the masses really not fall further to the center and why?

In the Big Bang Model, redshift was interpreted as a movement of galaxy systems caused by an expansion of space on a large scale. But whether the velocity increases from our point of view because the space expands and that only on a large scale, or whether the gravity of the total gravitation increases in the direction of the R_U, cannot be decided in this way.

What generally speaks against the idea of gravity is that there are no stable static solutions of Einstein's equations without, for example, a cosmological constant.

But for the Big Bang Model, numerous auxiliary conditions were also introduced and yet the theory does not fully explain the structure of our universe. Let's assume here, too, that the universe is designed in such a way that the radius applies $R_U = R_S$ and increases with c, then the Schwarzschild condition for black holes is followed by the fact that the mass of the universe increases constantly with R_U and distributes it evenly over each new larger sphere.

In order to meet the condition of isotropy our position must be relatively close to the center. In addition, the mass must be evenly distributed. This would mean that not only the total mass increases, but the total number of particles also increases and thus the individual mass of each particle becomes smaller and smaller.

This is a very large intervention, but it gives us a statically stable system again, which is becoming increasingly red shifted in the R_U direction and some new correlations, such as small back accelerations, which make dark matter superfluous.